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FORMER CONTRIBUTIONS TO "THE OPEN COURT."

EDWARD. C. HEGELER.

In No. 15, Mr. E. C. Hegeler explains his view of the soul. The soul, he says, is the form of a very complicated, self-acting mechanism of living substance; a part of its activity is accompanied with feeling; the feelings correspond in *form* to the most essential parts of the mechanism. The soul, as expressed by Bock, enters into our brain through the gateway of the senses. Reason is formed through the instrumentality of language. Noiré says: "Man thinks because he speaks." Immortality does not only mean the indestructibility of matter and energy but *soul preservation*. It implies not only continuance of life, but life in a special form. We can to a great extent renew ourselves by forming our soul in the growing generations through education and by example. To preserve and to elevate the *quality* of the human soul, that is the basis of ethics.

WHEELBARROW.

"WHEELBARROW" speaks to the laboring men from the standpoint of a laborer, although he does not work with a shovel and a wheelbarrow now. In his first essay, published in one of the early numbers of THE OPEN COURT, he says: "I sign my name 'Wheelbarrow' because that is the implement of my handicraft or was when I was a strong man. I was by profession a 'railroad man;' my part in the railroad business was making the roadbed by the aid of a pick, a shovel and a wheelbarrow." We quote this passage because from our personal acquaintance with "Wheelbarrow" we understand that it is literally true.

"Wheelbarrow" treats the labor question in a manner peculiarly his own, with illustrations drawn from every-day experience and presenting a moral which may be seen at a glance. He advises the working men in a friendly, persuasive way, and criticises many of their methods of reform as harmful to themselves, tyrannical and unwise. These essays have been much admired, not only by the working men, but also by men eminent in American literature. In an editorial article on "Wheelbarrow," the *Boston Herald* said: "He possesses in a striking degree the rare ability of being able to treat of complicated matters in so lucid and simple a manner as to make them easy of comprehension to those who have never before given the subject much thought. Last year he published a series of tracts on the labor question which were widely read, but not half so widely as they deserved to be. Treating of his subject from a working man's standpoint, he displayed an extraordinary wealth of apt but homely illustrations."

WHEELBARROW'S essays are contained in the following numbers of THE OPEN COURT:

Chopping Sand	page 353 in No. 13
The Laokoon of Labor	" 410 in No. 15
The Poets of Liberty and Labor. THOMAS HOOD	" 461 in No. 17
To Arms	" 615 in No. 22
The Poets of Liberty and Labor. Continued. GERALD MASSEY	" 745 in No. 26
Making Scarcity	" 901 in No. 34
Economic Conferences. I. A review of Geo. A. Schilling's lecture	" 950 in No. 37
Economic Conferences. II. A review of Lyman J. Gage's lecture	" 993 in No. 40
Economic Conferences. III. A review of T. J. Morgan's lecture	" 1104 in No. 47

PROF. E. D. COPE.

In Number 23 Professor E. D. Cope treats of "Evolution and Idealism," and finds in the evidence of evolution the refutation of the doctrine of idealism. He concedes that much of what we consider the objective world is unreal and has no existence, except, as we perceive it, like the snakes, for instance, to the victim of *delirium tremens;* and he agrees that "the properties of matter" owe much of their character, and even their existence, to our senses, which give matter all its qualities, or, at least, modify and change them according to the number of our senses and their ability to perceive. Yet, for all that, he says: "If a given supposed object be purely a mental state on the part of the subject, a rational cause for the production of that state is wanting."

With great self-confidence, Professor Cope says: "Evolution gives the *coup de grace* to idealism of the consistent type. In the gradual unfolding of organic life it sees the two universal facts, subject and object. It sees them interact and influence each other," and further on he says: "It is equally competent for the materialist to deny the existence of mind as for the idealist to deny the existence of matter. The materialist beholding the imperfection of the senses may pronounce them to be, one by one, incompetent witnesses, and declare them to be illusions. But the fact is, both exist, object and subject, matter and mind."

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A FLAW IN THE FOUNDATION OF GEOMETRY.*

BY HERMANN GRASSMANN.

Geometry, at the present day, still lacks a scientific beginning. The foundation on which the entire structure rests, suffers from a flaw that necessitates a complete reconstruction of the system.

In making an assertion such as the above, which threatens to overturn an edifice consecrated by the authority of centuries, it is incumbent upon me to support the same with the most conclusive kind of reasons.

The flaw, the presence of which I propose to show, is most easily recognizable in the concept of the plane. Taking the definition given in the systems of geometry with which I am acquainted, I find it to be assumed fundamentally therein, that a straight line which has two points in common with a plane falls wholly within the plane;—be it that this is tacitly accepted (as Euclid has done), or embraced in the definition of a plane, or propounded, finally, as a distinct axiom. The first case,—where the assumption is tacitly made,—is on its face unscientific; while the second, as I shall presently show, can with no more reason pretend to the requisites of scientific character.†

It is clear that the plane is already defined, be it as the aggregate of the parallel lines that can be drawn from a straight line in a direction not included in the latter, be it as the aggregate of straight lines that can be drawn from a point to a straight line. Adhering to the first definition now, it is clear that every straight line which cuts two of these parallels must also cut all the rest—a proposition not demonstrable without the assistance of a series of auxiliary propositions. If now a plane be defined as a surface which wholly contains all straight lines having two points in common with it, it is at once seen how, thus, the preceding proposition, hidden in this definition, has been smuggled into the domain of geometry; and just as little as any mathematician would permit an attempt to avoid the proof of the proposition that the opposite sides of parallelograms are of equal length, by defining a parallelogram as a quadrilateral whose opposite sides are equal and parallel; no more are we to allow the proposition stated above to be improperly introduced into geometry by a definition, like that, of a plane.

The only remaining course, therefore, in case we wished to hold to the method of geometry hitherto pursued, would be to convert that proposition into an axiom. But if an axiom can be avoided without having to introduce a new one in its stead, it must be done; even though it should bring about a complete reconstruction of the whole science. For, in this way, the science must gain substantially in simplicity.

Proceeding now from this defect, which we trust we have shown, and going back still farther, to discover the causes of the same, we will find the latter to lie in the faulty conception taken of geometrical axioms. In the first place, it must strike the attention, that by the side of veritable axioms which set forth geometrical intuitions, entirely abstract propositions are frequently presented under the same name, as: "if two magnitudes are equal to the same magnitude, then they are equal to each other," and which, if by axioms assumed truths be understood, are by no means entitled to be thus named. In fact, I believe I have shown before (Sec. 1, Theory of Extension), that the abstract proposition just cited expresses only the idea of equality; and this holds good, also, of the other abstract propositions which point out substantially that what is produced in the same way from the same is itself the same [equal].

From the imputation of confounding axioms with assumed concepts Euclid himself, however, is free; Euclid incorporated the former among his postulates (*αἰτήματα*), while he separated the latter as common concepts (*κοιναὶ ἔννοιαι*)—a proceeding which even on the part of his commentators was no longer understood, and likewise with modern mathematicians, unfortunately for science, has met with little imitation. As a matter of fact, the abstract methods of mathematical science know no axioms at all; the initial proof, in these methods, is brought about by the combination of predications;† use being made of no other law of pro-

* Translated by μκρκ from *Die Ausdehnungs-Lehre von 1844* oder *Die lineale Ausdehnungs-Lehre; ein neuer Zweig der Mathematik*, of Hermann Grassmann, pp. 32–37. Leipsic: 1878. Otto Wigand.

† As to the requisites of scientific character mentioned, see Grassmann's remarks at the close of editorial article, on page 1472 of this number.—Tr.

* It is quite possible, indeed, that an exposition exists, in which the defect animadverted upon has been avoided, without having come to my knowledge. But as the doctrines of parallels, that stumbling-block of mathematicians, would have had to have been cleared up in such an exposition, I could therefore assume with approximate certainty that no such exposition as yet exists.

† See p. 1370 of The Open Court, lines 11–23, first column.—Tr.

gression than the universal one of logic that that which is predicated of a series of objects so as to apply to each separately, can be predicated in fact of each separate object belonging to that series. To set up this law of progression as an axiom, which, as we find, embraces merely an act of reflection upon what was intended to be said by the general proposition, can occur to no mathematician; this is done, improperly, in logic; and sometimes even, it is attempted to be proved in that science.

In geometry, therefore, only those truths are left as axioms, which are derived from the conception of space. These axioms, accordingly, will have been correctly reached, if when taken together, they afford the complete conception of space, and if, further, none be set forth which does not aid in completing this conception. Just here the true cause of the defective foundation of geometry in its present shape appears. Thus, on the one hand, axioms are overlooked that express primary conceptions of space, and which, afterwards, when their application is demanded, must be tacitly assumed; on the other, again, axioms are laid down, that express no fundamental conception of space, and which, consequently, upon closer examination, turn out to be superfluous; while, taken together, the axioms in general afford the impression of an aggregate of the clearest possible principles, arranged with a view to the easiest possible methods of proof.

The axioms of geometry, in the form we are compelled to assume them, state, rather, the fundamental properties of space as originally given to our representative faculty; viz., its simpleness and relative limitedness.

The simpleness of space is stated in the axiom:

I. "Space is in all places and in all directions constituted the same; *i. e.*, in all places and in all directions the same constructions can be made."

This axiom, from its very form of expression, is separable into two axioms, of which one lays down the possibility of continued motion, and the other the possibility of a change of direction; *viz.*:

II. 1. "Equality is conceivable with difference of place."

2. "Equality is conceivable with difference of direction, and particularly when the direction is reversed."

Let us call those constructions "equal and concurring" [like-running*] which are made in exactly the same manner at different places, and thus are only distinguished with respect to place; those which are constructed in reversed directions in exactly the same manner, although in different places, "equal and contrary-running," in short, "reversed"; and those "exactly equal" which are different only with respect to place and direction. Adhering to these terms, further, to denote the results of the construction, we can express the above two axioms, with emphasis upon the subordinate clause of the second [II, 2], more simply thus:

III. 1. "That which is formed by equal and concurring [like-running] constructions, is likewise equal and concurring.

2. "That which is formed by reversed constructions, is, in its turn, reversed."

3. "That which is formed by exactly the same [exactly equal] constructions (although at different places and from different initial directions) is in its turn exactly the same [exactly equal]."

The first two of these three axioms constitute the positive assumption of that part of geometry which answers to the first part of our science.

The relative limitedness of space is set forth in the axiom:

"Space is a system of third degree."

The immediate obviousness of these axioms* and their indispensability must strike every person at once; without the first three [I, II, and III] there is no straight line, no plane, no angle possible; while the last sets forth space itself in its threefold extension. And although in the usual expositions they are for the most part passed over, yet it is not difficult to point to the places where they are tacitly employed. That they are adequate to the ends of geometry, can only be completely found out by the development of geometry from this germinal point.

* We adopt here more nearly the common way of conception, only substituting the more definite ideas of concurring [*gleichläufig*, like-running] and reversed [*gegenläufig* contrary-running], for the concept parallel.

* It may appear strange that the proposition is not taken here as an axiom, which states that between two points only *one* straight line is possible, or, as Euclid expresses it, that two straight lines cannot enclose a space (*χωρίον*). But this is contained in the first axiom, when correctly conceived; thus,—if two straight lines, which have one point in common, were to have still another point in common, then if the two lines did not at the same time have all other points in common and so completely coincided, the space at the second point mentioned would be quite differently constituted from what it was at the others. Should this proof, which moreover, would assume a stricter form in the actual development of the science, appear to bear too marked a philosophical stamp, we may nevertheless propound the proposition as a partial axiom for mathematical procedure, by merely keeping in mind its connection with the first axiom alone.

THE DREAM-BEECH.

FROM THE GERMAN OF RICHARD LEANDER BY *μκρκ*.

It is a hundred years or more ago since the lightning struck it and split it from top to bottom, and the plough now passes over the place where it grew. But in those days, upon the grassy knoll some few hundred paces from the first house of the village, a grand old beech-tree stood. And what a tree it was!—Such trees do not grow any more, for animals and people and plants and trees are all getting smaller and meaner every day. The country people used to say it came down from the time when the heathens lived in the

land; that a saint had been killed beneath the tree by the wicked men, and that its roots had drank of the holy blood, which ascended into the trunk and the branches and made it so great and strong. Who knows if it be true? But there was a great secret about the Beech. Everybody in the village, both great and small, knew *that*. Whoever fell asleep beneath it and had a dream—that dream was sure to come true. This was why it was called the Dream-Beech long before any one could remember, and nobody called it anything else. But there was something besides. Whoever lay down to sleep beneath the dream-tree durst not think of what he was going to dream. If he did, he dreamt of nothing but confusion and chaos that no sensible person could make anything out of at all. To be sure, it was a hard thing to do, because most people are altogether too curious for that, and so nearly everybody that tried it was sure to fail. Indeed, at the time when this story happened, I do not believe there was a single soul in the village, either man or woman, that had succeeded a single time. But all this about the dream-beech was true—that was certain.

One warm summer's day, when not a breath of air was stirring, a poor young apprentice came wandering along the high-road. It had gone hard with him for the many years he had passed among strangers, and as he came up to the village, he turned his pockets inside out to see what was left—but they were all empty. "What is to be done?" he thought to himself. "Worn out and weary; nobody will take you in, and 'fighting'—they have well called our struggle for bits of charity 'fighting'—that's a wearisome business." Just then he saw the lordly beech-tree on the grassy knoll before him, and as it stood but a few paces from the road, he threw himself upon the grass beneath it to rest himself a while. But the tree had a curious way of rustling, and as its branches swayed softly to and fro, there fell here and there through the leaves a ray of glittering sunshine, and here and there a particle of the blue sky beyond shone through upon him: his eye-lids drooped and he fell into a deep sleep.

The beech-tree dropped a branch with three leaflets, that fluttered and fell upon his breast. He dreamt that he sat in a cosy room at table, that the table was his and the room his and the house his too. A young woman stood before the table, resting her hands upon it and gazing kindly upon him, and that was his wife. And upon his knees he held a child which he was feeding porridge, and as the porridge was too hot, he was blowing into the spoon. His wife said: "What a fine nurse you'd make, dear!" and then she laughed at the idea. Another child, too, was jumping about the room, a stout, chubby-cheeked little fellow, with a big carrot on a string, which he was chasing about and shouting at, as if it were a real, live fox. And both these children were his.—This was his dream, and it must have pleased him quite, for, as he slept, his whole face was covered with smiles.

When he awoke it was almost evening. The village shepherd with his sheep stood before him, quietly working at his net. He sprang up refreshed, stretched himself, looked about, and exclaimed "Kind heaven to him whose lot should be as this! 'Tis nice at least to know how it really would be." Then the shepherd approached him and asked him where he was going and whether he had ever heard about the tree. When he was convinced that he was as innocent as the babe unborn, he cried out: "You're a lucky fellow? I know you dreamt something good; I could see it on your face. Why, I was looking at you all the time you were asleep!" Then he told him what the story about the tree was: "Your dream will come true; that is just as sure as that this is a ewe and that a ram. Just ask the people at the village if what I say is not so. And now tell me what you dreamed!"

"Old fellow," replied the apprentice with a merry laugh, "that's the way they pump country-folk. I'm going to keep my dream to myself; you won't take that amiss surely. But depend upon it, nothing will come of it after all!"

He did not say this for the sake of saying it; he really meant it, for as he went toward the village he kept saying to himself: "Fiddle-faddle, shepherd's prattle! I'd like to know where the tree got the knack from."

From the gable of the third house as he came into the village, a long staff jutted out into the street; a gilded crown hung from it, and just beneath, in front of the door-way, stood mine host of the Golden Crown. He was in the best of humor, for he had just had supper, and had had quite enough, and that was the best time of day for him. The apprentice civilly doffed his cap and asked if he would keep him over night for a "God bless you."

The landlord looked at the trim young fellow in his dusty and torn clothes, eyeing him closely from top to toe. Then he nodded kindly and said: "Sit right down in the arbor there, near the door. May be there's a bit of bread and a jug of wine left over, and in the meantime they will make you a bed of straw." Whereupon he went into the house and told his daughter.

The daughter brought the bread and the wine, seated herself opposite him and had him tell her of the strange lands and things he had seen. And she told him, in turn, what she knew of the village-gossip, of the wheat-crop, that their neighbor's wife had had twins, and when the next village-dance was to take place at the Golden Crown.

Suddenly she sprang from her seat, and bending over the table to the apprentice, she said: "What are those three leaves there in your jacket?" The apprentice looked and found the branch with the three leaves; it had stuck fast to his jacket. "Oh, they must be off of the big beech-tree just outside the village," he said, "I took a little nap under it."

The maid listened eagerly and waited for him to say more. But he stopped; and then she artfully began to ask him all sorts of questions, till she found out for sure that he had slept under the Dream-Beech; and then she went around and around just like a kitten about a plate of hot milk, till she felt sure that he knew nothing of the wonderful power and virtue of the Dream-Beech; for he was an artful rogue and made out that he knew nothing about it.

When she had got that far, she arose and brought him another jug of wine, plied him to drink of it, and told him all sorts of things that she had dreamed and of what a great pity it was that none of them had ever come true.

In the meantime the shepherd came back from the pastures, and was driving his sheep through the village street. As he was passing the Golden Crown, and saw the maid sitting in the arbor and talking so earnestly with the apprentice, he stood still for a moment, and said: "Yes, he'll tell *you* all about his pretty dream of course, and I could not get a word from him." And at that he went his way with his sheep.

But the apprentice would tell the maid of nothing like a dream and she grew so curious that she could stand it no longer, and she straightway asked him what he had dreamt while he was asleep under the beech-tree.

Now the apprentice was an arrant rogue and his dream had made him mischievous and happy, so he puckers up his face, winks slyly with his eye, and says: "The beautiful dream that I had! It must be true. Yet I wouldn't dare to tell what it was." But she kept on urging and teasing him to tell, when he moved up quite close to her and said gravely: "I dreamed that I would marry the daughter of mine host of the Golden Crown, and that one day I would myself be the landlord here."

At first the maiden turned as white as chalk and then as red as a rose; she went into the house, but after a while she came again and asked him if he was in earnest and if he had really dreamed that.

"Why, of course!" he said, "and the one I saw in my dream was just the image of you." Again the maiden went into the house, but this time she did not come again. She went into her little room, and the thoughts poured over her heart as does the water over a river-weir: ever afresh and anew, on and on, as if the end would never come. "He does not know about the tree. Wish as I may, it *must* come so; and nothing can help it." Thereupon she laid herself to rest and the whole night long she dreamed of the young apprentice. When she awoke the next morning, she knew his face by heart, for she had seen it so often in her sleep—and a trim fellow he was, there could be no doubt about that.

The apprentice had slept well and soundly upon his litter of straw; dream-beech and dream, everything he had said to the daughter of the landlord the evening before was long since forgotton. He was standing at the door of the tavern and was about to give his host his hand in parting. Just then the daughter entered, and as she saw him standing there ready to leave them, a strange feeling of dread came over her, a feeling as if she could not let him go. "Father," she said, "the wine has not been drawn off yet and the young fellow hasn't anything to do; if he would stay with us a day, he might earn his meals and a penny or two to help him on his way." The landlord of the Golden Crown had nothing to say against this, for he had just eaten a hearty breakfast and had had quite enough, and that was the best time of day for him.

But it was slow work with the wine. There was always this and that to be done, and the daughter always found something to bring the apprentice up from the cellar. When the great tun was empty and all the bottles filled, she said it would be just the thing for him to help in the fields a while; and when that was over with, there was all manner of things to be looked after in the garden—things that nobody had ever thought of before. And in this way week after week passed away, and every single night the maiden dreamed of the apprentice. In the evenings she would sit with him in the arbor before the house, and when he told of his mishaps and misfortunes among the strange peoples he had visited, a gnat or midge would always get into her eye and she would have to rub it out with the end of her apron.

A year had passed away, and the apprentice was still at the house of the landlord of the Golden Crown; everything was scoured and polished, white sand had been strewed upon the floors, and ever-green palm branches scattered about the house; the whole village was keeping holiday. For to-day the travelling apprentice was to marry the daughter of the landlord of the Golden Crown; and if any one, because of envy, did not rejoice in heart, he at least made everybody believe that he did.

* * *

Not long after, as the landlord of the Golden Crown sat quietly smoking in his great arm-chair, he fell into a deep and peaceful sleep; and as he did not awake again, they came to rouse him. But his eyes had closed forever.

The young apprentice had to take charge of the inn, and really did become landlord as he had said in jest, and everything else turned out just as he had dreamed it under the old beech-tree. For the two children soon came, and I suppose he must have taken one of them in his lap some time or other and fed it, and blown into the spoon too, and of course the other one ran about the room with the carrot, although the man I got this story from, did not say that he did and I had really forgotten to ask him about it especially. But it must have been so, for everything that people dreamed of under the old beech-tree came out to a dot.

One day—it might have been four years after the day of the wedding—the young landlord of the Golden Crown was sitting in the public-room of the inn. His young wife came in and going up to him, said: "Just think, love; yesterday noon one of our harvest-men fell asleep under the Beech-Tree. Guess what he dreamed? He dreamed he was rolling in gold. And it was foolish old Caspar too, that everybody is so sorry for, and whom we keep only out of pity. What will he do with all his money?"

Then the husband laughed and said: "How can you believe all that silly nonsense? You are so sensible in other things. Just think for a moment whether a tree can know anything about what is going to happen, and if it be ever so old and so beautiful."

The wife looked at her husband aghast; then she shook her head and said gravely: "Husband, do not scoff. You should not jest about such things."

"I am not jesting, wife!" said the husband.

At that the wife was silent for awhile, as if she did not understand him quite, and then she said: "O, why do you say so! I should think you had every reason to be thankful to the blessed old tree. Has not everything come, as you dreamed it?"

When she said this, the husband made the pleasantest face in the world and replied: "God knows that I am thankful—thank Him and thee. It was indeed a pretty dream. It is as if it were but yesterday, so well do I remember it. And a thousand times more beautiful than in my dream has everything come out; and a thousand times more sweet and beautiful art thou, my love, than the wife I saw in my dream beneath the beech-tree.

Again the wife looked at her husband with open eyes as he went on: "But as to the tree and the dream I dreamt, my sweet one—why, surely it was nothing wonderful for me to dream once of happiness, when I had fared so ill among the strange peoples I had visited."

"But you dreamed that you would marry me though!"

"I never dreamed that! It was only a young woman with two children that I saw, and she wasn't near so pretty as you are, nor the children either."

"Fie!" cried his wife. "Are you going to disown me, or the tree? Didn't you tell me the first day I saw you—it was in the evening out in the arbor—didn't you tell me you dreamt you would marry me, and one day become landlord of the Golden Crown?"

The husband thought, for the first time since, of the jest of that evening, and he said: "It can't be helped, dear wife! Really I didn't dream of you then; and when I said I did, it was only in jest. You were so inquisitive and I wanted to tease you."

His wife broke out into bitter tears and went out into the open air. After awhile he went after her. She was standing by the well in the court-yard and was weeping still. He tried to comfort her, but in vain.

"You have stolen my love and robbed me of my heart," said she. "I shall never again be happy."

Then he asked her if she did not love him, love him as she loved none other on earth, and whether they had not lived contentedly and happily together, as had none other in their village. She said yes to it all, but still she was as sad as before, despite all his comforting.

So he said to himself: "Let her have a good cry; other thoughts will come to her over night; she'll be the same as ever to-morrow." But he deceived himself; for on the morning after, his wife did not cry any more it is true, but she was solemn and sad, and tried to avoid her husband. It was of no use to try to comfort her; every attempt failed, as it did the evening before. The best part of the day she sat brooding in the corner, and started with fright if her husband approached her.

When this had lasted several days without any change whatever, a great sorrow took possession of him too, for he feared that he had lost the love of his wife forever. In silence he walked the house, thinking and thinking of what could be done, but nothing seemed to come to him. So one day at noon-tide he walked beyond the village and sauntered through the fields and meadows. It was a hot day in July; not a cloud in the skies. The ripened corn surged like an ocean of gold and the birds were singing merrily; but his heart was laden with sorrow. In the distance he could see the old beech-tree standing: like a monarch among trees, it towered high into the heavens. It seemed as if it were nodding to him with its great green branches, were calling to him as to a friend of long ago. He went and seated himself beneath it; and thought of the times that had passed. Five years had gone by since the poor apprentice had first rested beneath its branches and dreamed so wondrously. How beautiful! And the dream had lasted for five years —and now? All was over!—All over?—Forever?

The beech-tree began to rustle, as it had rustled five years before, and again it swayed its mighty boughs. And just as before, there fell here and there through the waving leaves rays of glittering sunshine, and here and there a particle of the blue sky beyond shone through upon him. His heart was quieted, and he fell asleep; for he had not slept for worry during the past few nights. And it was not long before he dreamt the dream of five years before; but now the wife at the table and the children playing about it were the dear familiar faces of his wife and his children. And his wife was looking at him so kindly—so kindly!

He awoke, but when he saw that it was only a dream, he grew sadder still. He broke off from the tree a little green branch, and went home, and placed it in the hymn-book.

The next day—it was Sunday—as his wife was about to go to church, the branch from the beech-tree fell out. At this her husband, who was standing near her, blushed, and stooping down to pick it up, he tried to stick it in his pocket. But the wife saw it and asked him what the leaf was.

"It is from the Dream-Beech; it wishes me kinder fortune than you do!" said the husband. "For yesterday, as I was sitting beneath it, I fell asleep. It wanted to comfort me; for I dreamed that you were again kind to me and had forgotten all. But it is not true! There is no virtue in the good old Beech. A great and beautiful tree it is, but of the future it knows nothing."

At that his wife stared upon him, and then the sunshine played in her face: "Husband, did you, did you really dream that?".

"Yes," he answered firmly, and she saw that it was the truth, for his face quivered in keeping back the tears.

"And I was really your wife?"

When he said yes to this too, the wife fell upon his neck and kissed him so often and so fervently that he could not resist her. "Now God be praised," she said, "for all is well again! I love you so, indeed I love you as you cannot know. And how I've suffered from the dread that God might not want me to love you and that perhaps He had wanted me to marry some other man! For you did steal my heart, you bad man, and you did cheat me a little. But now I know that it didn't do you any good, for it would have turned out so anyhow."

At this, she was silent a moment, and then she said again:

"But you will not say anything bad about the Dream-Beech again, will you?"

"No, never; I believe in it now. May be not the same way that you do, but just as firmly though. Trust me!"

"So let us fasten the branch in the front part of the hymn-book. We must not lose it."

THE OLD AND THE NEW MATHEMATICS.

In mathematics, just as in all sciences and in religion, we have an orthodoxy sanctioned by the authority of many centuries. This orthodoxy represents a conception of things, which in the past, to some extent, has proved sufficient for our needs. It is presented in the most direct, and for its purpose therefore in the best method — namely in the shape of dogmatism. Thus matters are, we are told, and it suffices to know that they are so. Dr. Brooks says: "Shall we not be content and let 'well enough alone' and rest with absolute certainty on the deduction of the present system of geometry?"

The representatives of orthodoxy are opposed by a class of heretics, who claim that humanity would have progressed more rapidly but for the impediments of dogmatism. The ideas of dogmatism, they say, are fundamentally erroneous, and must be overturned. Room must be made for doubt. Humanity, up to the date of the appearance of heretical views, it is held, has been erring under the dominance of orthodoxy, and we must commence to live the life of mankind over again.

These heretics, tearing down and criticizing the old dogmatism, are by no means useless, or nefarious, or dangerous men, although they are very often looked upon as acting the *rôle* of Mephistopheles and although, as a rule, they exhaust their power in mere negations without being able to build anew. Voltaire said: "If God did not exist, we should invent him." Similarly we can say: "If the devil did not exist, we should invent him." "The spirits who deny" play a very important part in the household of nature.

> "Man's aspiration flagging seeks too soon the level,
> Unqualified repose he learns to crave;
> Whence, willingly, the comrade him I gave
> Who works, excites, and must create as Devil."

The negative criticism of heresy leads the orthodox conception to a higher plane of development, not by tearing down, but by forcing us to remould it, to eliminate its errors, and thus to unify its tenets with the other facts of reality. If we really had to commence to live the life of humanity over again, we would again have to start with the old or a similar dogmatism, until we were sufficiently matured to enlarge our views to a broader conception, in which our former orthodoxy is not so much destroyed as outgrown.

Dr. Brooks represents the orthodox standpoint of mathematics. He dogmatically believes in the finality of mathematical axioms; he says: "To know how we know the axioms to be true would be equivalent to proving them to be true." But he does not believe that we can know this *how*. "There is no 'how,'" he says. * * We just know that they are true and tha

is the end of it. * * To prove a truth is to establish it by some other truth; but there are no truths back of or before these axiomatic truths which authenticate them. They are absolutely first truths, underived and self-existent, and as such are cognized by the mind."

This standpoint of orthodox dogmatism in mathematics may be called the intuitive method. In opposition to it John Stuart Mill proposes his heterodox views, which are best characterized as the empiricist method. Mr. Mill says in his System of Logic (2, V. Sec. 1):

"The points, lines, circles, and squares which any one has in his mind, are (I apprehend) simple copies of the points, lines, circles, and squares which he has known in his experience. The idea of a point I apprehend to be simply our idea of the *minimum visibile*, the smallest portion of surface which we can see. A line as defined by geometers is wholly inconceivable."

If Mr. Mill's empiricism were correct, mathematics would be an experimental science, like chemistry and the other natural sciences. There would be no difference between formal sciences and experimental sciences, and such things as necessity or necessary truths would be illusions. Mr. Mill accepts this consequence and tries to eliminate "necessity." He says:

"This character of necessity, ascribed to the truths of mathematics, and (with some reservations to be hereafter made), the peculiar certainty, attributed to them, is an illusion. * * *

"When, therefore, it is affirmed that the conlusions of geometry are necessary truths, the necessity consists, in reality, only in this, that they correctly follow from the suppositions from which they are deduced. Those suppositions are so far from being necessary that they are not even true; they purposely depart, more or less widely, from the truth. The only sense in which necessity can be ascribed to the conclusions of any scientific investigation, is that of legitimately following from some assumption, which, by the conditions of the inquiry, is not to be questioned."

According to Mr. Mill, our mathematical conceptions "are not even true; they purposely depart, more or less widely, from the truth." They certainly would depart from the truth if mathematics were an experimental science, if mathematical lines were images of material lines, perhaps of lead-pencil lines, if mathematical points were truly a *minimum visibile*, etc. Mathematical concepts depart from the real diagrams which we draw for the purpose of assisting our mathematical imagination, but they do not, therefore, depart from the truth.

If Mr. Mill's theory were correct, if mathematics were not a creation of pure formal thought, invented for properly comprehending the laws of pure form, if it were based upon the inaccurate, unreal, and, therefore, untrue images of material points, lines, circles, planes, etc., we would have to remodel the whole science of mathematics so as to make our conceptions of points and lines and planes "true." But an experimental mathematics of that kind, it need not be said, would lose all its value, its certainty, and its exactness. Indeed, as a system of purely formal laws, it would be "untrue"; for it would conflict with the principle of mathematical conceptions that limits the field of mathematics to pure forms and excludes from it any kind of material existence.

The basis of mathematics is pure formal thought. The pure form of a thing is the spacial relation of its parts among themselves. The pure form of a leaden ball is its globular shape. Mathematics, accordingly, deals with the laws of spacial relations purely, without taking into consideration anything else. All other qualities, especially those relating to matter and force, are rigidly excluded.

Dr. Brooks says: "Some things not only exist but their existence is a necessity. They exist independently of all conditions and are subject to no contingencies." Among these things, time, and space, and axiomatic truths are classed. The paper, he says, "has length, breadth, and thickness; length, breadth, and thickness are possible only in space, therefore space also exists."

Certainly space exists, but it does not exist of itself. It has no absolute existence. It exists as a property of reality, and our conception of space has been abstracted from reality. 'Length, breadth, and thickness,' we propose to say, '*are* space.' If we say with Dr. Brooks, they "are possible only in space," the dualistic error is near at hand, that space is not a mere abstract idea representing the quality of extension abstracted from extended things, but that it is something existing of itself; something which is the condition of extension, which makes it possible that things can have length, breadth, and thickness.

Space being an abstract and not a thing of itself has been supposed by some philosophers to be a non-entity. Descartes says,* that if that which is in a hollow vessel were taken out of it without anything to fill its place, the sides of the vessel, having nothing between them would be in contact. This is erroneous. Space is not a non-entity, but a real property of things. The spacial relation between two sides of a hollow vessel remains the same whether there is or is not any matter between them. If we could succeed in annihilating the whole world, all spacial relation would be destroyed with it. But let there be one atom only, or one given point, where in our imagination we may place ourselves, and we will therewith establish a possibility of motion in all directions, and the possibility of constructing in our imagination other points in different distances or relations: we would have space—not a part of space, but space entire. Space being the possibility of motion, is determined by measurable relations, in which existences or possible existences or points can be arranged. A part of

* Princip. Phil. II. 18.

space, alone and absolute, can neither be created nor can it be annihilated; for space being of itself a mere possibility of relations, is always entire. Thus the minutest part of a parabola contains the law of the whole parabolic curve into infinity, and so with the slightest part of space the whole of space is determined.

The old orthodox view of mathematics takes its stand on axioms (such as "a straight line is the shortest distance between two points"), which are accepted as self-evident truths. Among the simplest mathematical theorems is one stating that "the corresponding angles of parallels cut by a straight line are equal." Since an exact proof of this theorem was impossible, it has found a place among the axioms, and is in our textbooks usually treated as such.

Some mathematicians, however, did not rest satisfied with this solution of the Gordian Knot in the fashion of Alexander, and attempted to develop a geometry in which the theorem of corresponding angles should not be accepted as an axiom. They succeeded in establishing a new kind of geometry which was different from Euclid's geometry. Two straight lines cannot inclose a space according to Euclid; but in the new geometry, two straightest lines, if sufficiently prolonged, can inclose a space. To distinguish them from Euclidian "straight lines," it has been proposed to call them "straightest lines," both (straight as well as straightest lines) being the shortest possible distance between two points.

This new geometry has been called that of curved space, and further investigations* showed that there are two kinds of curvature, the positive and the negative. The positive may be represented as the convex surface of a globe and the negative as the concave surface of a hollow globe. The Euclidian theorems now appeared as special instances of this geometry. They can be considered as constructed in a plane the curvature of which is zero.

We learn from the attempts made in this direction that the mathematical axioms are by no means "absolutely first truths, underived and self-evident." They depend upon the special condition that the space curvature is zero, which (however justified for practical purposes) has been tacitly assumed.

* * *

We can generalize the concept space and consider the line as a space of one dimension, the plane as a space of two dimensions, and actual space as a space of three dimensions. It is impossible to form any intuitive conception of a space of four and, still less, of more than four dimensions. Nevertheless, we can abstract from dimensions altogether and conceive of such absolute space as 'Form, pure and simple.' In doing so we can lay down the laws which are equally valid for all kinds of spaces, whether of three, or four, or n dimensions. Algebra, indeed, is an abstraction of that kind, and algebraic laws are equally valid whether their symbols indicate lines, or planes, or solid bodies, or other things, as for instance logical concepts.

The ultimate step which can be taken in this direction is that of establishing a "theory of pure forms," as has been done by Grassmann. Grassmann recognizes no axioms whatever. He builds his "system of pure forms in general" and finds that Euclid's geometry, as well as the actual space of three dimensions, are special cases only of innumerable other possibilities, the laws of which are all contained in his "theory of forms in general." What Euclid called axioms are a few characteristic features which can be derived from the supposition that plane geometry is a system of second degree. Far from being first, or absolute, or independent truths, the axioms depend upon this supposition, and are applicable only for cases where it is avowedly accepted or at least tacidly assumed.

Grassmann no longer stands alone in the position he has taken; he has found followers who more and more realize that he has been the pathfinder of a new and fertile field of mathematical investigation. The ultimate basis of mathematics is no longer the intuition of space but the conception of "abstract form in general." The apriority of the mathematical laws of actual space has to be limited to the extent, that we can know by experience only that actual space has three dimensions, and we have learned to consider the world-space as one actual instance among many theoretical possibilities: it is a formal system of third degree.

Actual space, abstracted from reality, is a quality of real things representing their relations, the relations of their parts, and the possible directions of their motion. But actual space, as we can ascertain by experience, is at the same time a system of third degree. As a system of third degree, it is a creation of our mind, it is purely formal thought, to which all the rigidity and universality of formal laws is attached. The sentence "space is a system of third degree," is as little tautological, or begging the question, as that the earth is a spheroid; and it is at the same time just as much a matter of experience. The laws of a system of third degree apply to actual space with the same necessity as the principles of mathematical geography apply to the earth.

* * *

Dr. Brooks says: "Some truths are not only true, but they are necessarily true," and "the mind has the power of knowing that they are necessarily true."

That gunpowder explodes is true; but it is not necessarily true. In damp weather it may not explode; the explosion depends upon certain conditions. But if all the conditions upon which, according to our

* We mention the names of Lobatschefsky, Bolyai, Gauss, and Riemann.

experience, the result is contingent are fulfilled, we assume that it will explode. It ought to and very likely it will; but must it necessarily explode? Certainly not! There may be one condition which in all former cases was always fulfilled without our knowing it. If this one condition were absent in an eventual experiment the usual result would *not* take place. The results of experimental sciences are never necessary in this rigid sense. Rigid necessity does not depend upon conditions; it is intrinsic and we must be able to verify it as a necessity; we must know why or how it is a necessity, not by intuition, but by proof.

All formal truths are rigid necessities. Propositions, as for instance $2 \times 2 = 4$, and $(a + b)^2 = a^2 + 2ab + b^2$, possess intrinsical truth; for they do not depend upon external conditions, and hold good everywhere and for all possible cases.

For the sake of distinction, the truths of purely formal thought are called *correct*, and the truths of a well-ascertained experience *real*. Correct, accordingly, signifies that which is true in a mere formal sense, and real (in this limited sense) signifies that which has a material existence. Mr. Mill, therefore, in the above quoted passage, should have said that the mathematical suppositions are not realities (*viz.*, realities in the limited sense). They are not material existences. But that is no reason for declaring that they depart from the truth. If they are but correct, they are true; they are true so far as their form is concerned. By correctness we cannot gain substantial knowledge of things, but the correctness of our formal thought alone can afford that necessity, by means of which any kind of truth is established. Without the assistance of arithmetic, mathematics, mechanics, and logic, scientific knowledge cannot be obtained.

The assumption of Dr. Brooks that there are necessary truths, of which the mind has the power of knowing by intuition that they are necessarily true, would lead us back to the conception of "innate ideas." If we are not bound to explain why or how certain ideas are true, there is no means of discriminating between inveterate or inherited errors, and genuine truths.

The existence of the material universe is by no means necessary; nor is it necessary that actual space has three dimensions. We can imagine that we did not exist and that the whole world did not exist; we can imagine that a world existed, the space of which would possess two dimensions. But we cannot think it possible that $2 \times 2 = 5$; and we can positively understand why the laws of form in general must hold good under all conditions and in all possible worlds. If they were never realized in actual existences, they would nevertheless remain what they are—correct.

* * *

In the province of mathematics we move in an atmosphere of abstract thought. The simplest mathematical conceptions are by no means so absolutely simple as they appear; they are simple only in comparison to other mathematical ideas, to definitions of, and theorems about, complex figures. A bright little boy of six years may have very clear conceptions as to dogs, horses, and even engines or other concrete things, but there is little probability of his understanding the meaning of a mathematical point. That simple idea is too complex for his immature comprehension.

Dr. Brooks says:

> "A derivation of one truth from one or more other truths is called reasoning. * * * All reasoning can be traced back to truths which cannot be derived from other truths, and hence are not the result of reasoning."

According to our view the basic conceptions of mathematics and the axioms so-called, *are* the result of reasoning. They are not first truths from which we start quite from the beginning; they are not self-evident in the sense that there are no truths back of or before them; but we acquire them after a long exercise in mental work only. They are based upon a well-directed and disciplined discrimination. This discrimination between form and matter, simple though it appears to us now, is most subtle, and its importance is invaluable. It enables us to construct systems of, and to evolve the laws pertaining to, formal thought. This discrimination between form und matter is, therefore, the commencement of a higher development; it makes scientific thought possible.

The correctness of formal knowledge was formerly based on axioms which had to be taken on faith. But as long as the certainty of axioms is based upon intuition, mathematics (and all other formal sciences) must appear to hover in the air and have no connection with the solid facts of reality. Mathematicians, it is true, rarely were inclined to foster mystic views (Cabalistic and Neoplatonic mathematicians are exceptions), and Dr. Brooks also repudiates any kind of mysticism. Nevertheless as long as a science is ultimately based on intuition, there is room for any degree of mysticism. Grassmann's broader conception of mathematics has made all mysticism impossible. He has taught us to dive down to the bottom of the problems, where we can understand the origin and whole growth of mathematics and where they are seen to be in connection with the other facts of reality.

* * *

For our present purpose we are satisfied with having pointed out the connection which obtains between mathematics and the other facts of reality; but we may add for those interested in the philosophy of mathematics, that from Grassmann's standpoint the connection, also, that exists between the different mathematical theorems and solutions is more readily under-

stood. Hamilton's quaternions and the significance of imaginary quantities have been anticipated by Grassmann and appear in their connection with his system in a new light. Grassmann's method allows a survey of the whole field and thus gives to the student that easy freedom which a traveler feels who constantly keeps in sight the point towards which he is journeying, as well as the road on which he approaches it.

Grassmann says*:

"Since both mathematics and philosophy are sciences in the strictest sense of the term, the methods employed in each must accordingly have something in common, which gives them their peculiar scientific character. Now, we give a scientfiic character to a method of treatment when the student, on the one hand, is of necessity led by it to the recognition of every single truth, and on the other hand is placed in a position wherefrom he is enabled, at every point in the development, to survey the course of further progress.

"The indispensableness of the first requirement, *viz.*, *scientific rigidity*, every one will admit. As to the second, the same remains a point that is not as yet sufficiently recognized by the majority of mathematicians. Demonstrations are frequently met with, where, unless the theorems were stated above them, one could never originally know what they were going to lead to; here, after one has followed every step, blindly and at haphazard, and ere one is aware of it, he at last suddenly arrives at the truth to be proven. A demonstration of this sort, perhaps, leaves nothing more to be desired in point of rigidity. But scientific it certainly is not. The second requisite is lacking — namely, *the power of survey*. A person, therefore, that goes through such a demonstration, does not attain to an untrammelled cognizance of the truth, but he remains—unless he afterwards, himself, acquires that survey—in entire dependence upon the particular method by which the truth was reached. And this feeling of constraint which is at any rate present during the act of reception, is very oppressive for him who is wont to think independently and unimpededly and who is accustomed to make his own by active self-effort all that he receives. If, however, at every point in the development, the student is put in a position to see at what he is aiming, he remains master of his material, he is no longer bound to the particular form of presentation, and his assimilation of what he attains becomes actual reproduction." P. C.

"THE MARRIAGE PROBLEM."

BY LOUIS BELROSE, JR.

Faith is the substance of things hoped for, the evidence of things not seen.—HEB. XI. 1.

LET us have faith. The substance of things hoped for
Outlasts the shadow of the things not seen.
Let us have faith. The Word the worlds were framed by
Framed our ideals and will keep them clean.

Let us have faith. That Word, all hearts shall hear it.—
For us the time is long, but time is short.—
Let us have faith, and thereby, like the elders,
Obtain for recompense a good report.

Let us have faith, despite the mind's invention,
To hold the way that man and brute have trod.
Let us have faith in their unfailing instinct
Who made the virgin mother of a God.

*Grassmann, "Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik," Introduction, page xxxi.

SONNETS OF WINTER-TIDE.

MARY MORGAN (GOWAN LEA).

I.

THROUGH night's dark hours the snow fell, feathery light,
Until at break of morn, afar and near,
No leaflet is descried nor brooklet clear,
So close the earth is wrapped in mantle white.
Round yonder hill the snow-plough comes in sight,
Disclosing where the high-road should appear,
Tossing the white wreaths o'er the landscape drear,
Like waves before a ship, to left and right.
Symbolic picture of the march of mind—
Agent invisible—which yet doth know
Earth's proud wealth to dispose—philosophy
And art, and poesy—what it shall find
Upon its course as forward it may go,
New vistas opening for humanity!

II.

"Death is perhaps the last superstition."—Heine.

I READ the secret of the earth and air,
Concealed—revealed—in frozen twig and leaf,
As surely as in Spring-tide's blossom brief:—
Death opens Life's door, alway, everywhere.
The bud we call 'To-day,' fresh, fair, and sweet,
Roots in dead Yesterday, and fades from view
Yielding To-morrow—Mount of purple hue
Whose peak aspires the highest heaven to greet.
Decay and growth! A prophecy is each,
One of the other, in unbroken chain.
Distrustful human heart, how shalt thou reach
The knowledge that they are but aspects twain
Of the imperishable Substance—Life!
This truth attained, O soul, shall end thy strife!

MONTREAL, February 4, 1889.

THE LOST MANUSCRIPT.*

BY GUSTAV FREYTAG.

CHAPTER XXX.—Continued.

But the spirit of torment that had quartered itself under the roof of the castle did not confine its tricks to the Court and its household: it ventured to disturb the Professor also in his learned work.

Ilse was sitting alone, looking absent-mindedly at the pictures of Reynard the Fox, when the lackey threw open the door, announcing:

"His Highness, the Sovereign!"

The Sovereign glanced at the picture in the open book.

"So that is the view you take of our position. The satire of those pages is bitter, but they contain imperishable truth."

Ilse closed the book, coloring.

"The ill-behaved beasts are rude egotists; it is otherwise among men."

"Do you think so?" asked the Sovereign. "Those who have had experience with them will not judge so leniently. The two-legged animals that pursue their aims at the courts of princes are, for the most part, as reckless in their egotism, and as much inclined to

*Translation copyrighted.

profess their attachment. It is not easy to restrain their pretensions."

"Amongst the bad there are surely some better, in whom good preponderates?" rejoined Ilse.

The Sovereign inclined his head civilly.

"He who has to watch all keenly feels the narrow-mindedness of every individual, for he must know where and how far he can place confidence. Such an observation of various natures, which is always seeking to separate the reality from the glitter, to sound the worth of different characters, and to retain for the observer superior judgment, sharpens the perception of the deficiencies of others. It is possible that we may sometimes judge too severely, while you, with your warm feeling, fall into the amiable weakness of viewing men in too favorable a light."

"My lot, then, is happier," exclaimed Ilse, looking at the Sovereign, with honest commiseration.

"It is sweeter and happier," said the latter with feeling, "to give one's self up without restraint to one's feelings, to associate innocently with a few whom one chooses freely, to avoid by slight effort the ill-disposed, and to open one's heart gladly, and without restraint, to those one loves. But he who is condemned to live in the cold atmosphere of business, struggling against countless interests which clash together, can only carry on this existence by surrounding his daily life with regulations which will at least preserve him from overwhelming burdens and annoyances, and compel the foxes and wolves to bend their stubborn heads. Such rules of Court and government are no perfect work; there will often be complaints against them. You, perhaps, may have had occasion to remark that the customs and etiquette of a Court are not without harshness; yet they are necessary, for it makes it easy to us to withdraw and keep within ourselves, and maintain a certain isolation, which helps us to preserve our inward freedom."

Ilse looked conscious.

"But believe me," continued the Sovereign, "we still are human beings; we would gladly give ourselves up to the impulse of the moment, and live without restraint with those whom we esteem. We must often sacrifice ourselves, and we experience moments when such sacrifices are very severe."

"But within the princely family itself these considerations do not apply," exclaimed Ilse. "The mutual intercourse of father and children, brothers and sisters,—these holy relations can never be disturbed."

A cloud came over the countenance of the Sovereign.

"Even they suffer in their exposed position. We do not live together; we see each other less alone, generally under the observation of others. Each has his special circle of interest, is influenced by those about him, who perhaps diminish his confidence in his nearest relations. You know my son; he has all the qualifications of a good, open-hearted man, but you will have observed how suspicious and reserved he has become."

Ilse forgot all caution, and again felt a little proud of being a confidante.

"Forgive me," she explained; "I have never found that. He is only bashful, and sometimes a little awkward."

The Sovereign smiled.

"You lately expressed an opinion with reference to what would be advantageous for his future. That he should for a time become acquainted with the management of a large family estate; it would undoubtedly be good for him to learn the work of a country gentleman by experience. Besides this, he is not happy at Court."

Ilse nodded.

"Have you also remarked that?" asked the Sovereign.

"I will give good advice for my Prince," thought Ilse, "even if it is not quite agreeable to him. May I venture to say," she said aloud, "that this is the best time of all. For he must learn, your Highness, the spring tilling, which is in full operation, so there must be no delay."

The Sovereign was much pleased with this zeal.

"It will not be easy to find a place," he said.

"Perhaps your Highness has an estate in the neighborhood where there is a small manor-house."

"Then he could come often to the city," replied the Sovereign sharply.

"That would not do," continued Ilse, eagerly. "He must first thoroughly know the work of the people, and for that be constantly in the fields."

"I could not find a better adviser," said the Sovereign, in excellent humor. "There is nothing in the vicinity that will answer; I have thought, however, of your father's estate."

Ilse started with surprise.

"But our mode of life is not adapted for the accommodation of a prince," she replied with reserve. "No, gracious Sovereign, the domestic arrangements of our family would not be suitable to the pretensions of the young man. I say nothing of other considerations which formerly never occurred to me, and which have first come home to me here. Therefore, if I may speak what I feel, I am of opinion that this, for many reasons, will not answer."

"It was only a thought," replied the Sovereign, good-humoredly. "The object may perhaps be attained without encroaching upon your father. It has been my wish," he continued, with chivalrous politeness, "to give you and your father a public proof of

my esteem. I have special reasons for it." He looked significantly at Ilse, and she thought of the birthday of the Princess.

"I know the reason," she said softly.

The Sovereign drew his chair near.

"Your father has a large family?" he asked. "I have a vague recollection of having seen several rosy-cheeked boys about."

"They were my brothers," said Ilse, laughing; "they are handsome little fellows, gracious Sovereign, if I, as a sister, may praise them; they are at present somewhat uncouth, but good and clever. My Franz wrote to me only yesterday to beg me to greet your Highness for him. The little urchin thinks it is the right thing. Now, as I have the opportunity, I will show you the letter as he has written it; it is a stupid, childish message, but it comes from a good heart."

She felt in her pocket and brought forth a letter written in fair characters.

"See, your Highness, how well the child writes. But I must not show you the letter, for your Highness would find in it a confirmation of your opinion, that men have always selfish wishes in the background when they think of their princes. The poor boy also has his wish."

"Then let us have it," said the Sovereign.

Ilse showed him the letter; the Sovereign graciously took hold of the letter, and in doing so, his hand rested on hers.

"He is so barefaced as to ask your highness for an india-rubber ball. The ball is already bought."

She jumped up and brought a gigantic colored ball.

"This I shall send to him to-day, and I shall write to him that it is not seemly to beg of so great a personage. He is nine years old, but still very childish —your highness must forgive him."

Enchanted by this frank open-heartedness, the Sovereign said:

"Write to him, at the same time, that I wish to tell him he must endeavor to preserve through the dangerous paths of life the pure feeling and loyal spirit of his eldest sister. I also feel how great is the blessing of your character to all who have the happiness of breathing your atmosphere. In a course of life which is filled with harrowing impressions, in which hatred and suspicion take more from the peace of the soul than hours of repose can restore to it, I have still retained my susceptibility for the innocent freshness of a mind like yours. You give me genuine pleasure."

Again he laid his hand gently on hers; Ilse looked down confused at the praise of her dear Sovereign.

A hasty step approached; the Sovereign rose, and the Professor entered. He bowed to the Sovereign, and looked surprised at his wife.

"You are not ill?" he exclaimed. "Pardon, gracious Sir, I came in great anxiety about my wife. A strange boy rang the bell at the Museum, and brought a message that I must go immediately to see my wife, as she was ill; fortunately it was a mistake."

"I am thankful for the error," replied the Sovereign, "as it gives me the opportunity of saying to you what I was intending to mention to Madame Werner; orders have been given at the stable that a carriage shall be ready for you at any hour that you wish to take a journey in the neighborhood to pursue your mysterious investigations."

He took leave graciously.

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The Sovereign opened the window of his study; the air was sultry, the sun had been shining long upon the earth; now it had vanished, heavy clouds rolled themselves, like great shapeless porpoises, over the city and castle. The Sovereign fetched a deep breath, but the heavy, sultry air forced the smoke from the chimneys of the castle down to his window, enveloping his head like a great mist. He hastily opened the door of the gallery which led to the reception-rooms, and walked out. Against the walls hung a row of oil pictures, the portraits of beautiful ladies whom he had once favored with his attentions. His look strayed from one to the other; at the end of the row was an empty place; he stopped before it, and his fancy painted a picture with blonde hair, and a true-hearted, frank light in the eyes, more touching than any of the other faces.

"So late," he said, to himself. "It is the last place and the strongest feeling. They are fools who tell us that years make us indifferent. If I had come across her at the other end," he glanced back along the gallery, "at the beginning of my life, when I yet looked longingly at the roses on the cheeks of maidens and was touched by the song of hedge-sparrows, would such a woman then have preserved in me what I have lost forever? Useless thoughts of the past! I must in the present keep firm hold of what has come within the reach of my hand. She is indifferent about the weak youth; but she feels herself uneasy here, and if she tries to escape me I have no power to keep her back. I remain alone; daily the same wearisome faces, whose thoughts one knows before they are spoken, whose wishes one knows before they open their mouths, and whom one sees to be prepared with feigned feelings. Whatever wit or will they have works secretly against me; what I receive from them is only the artificial glitter of life. It is sad to be a master before whom living souls turn into machines, and year after year to open the lid and examine the works. I myself have made them," he said, jeeringly, "but I am weary of my work."

(To be continued.)

BOOK REVIEWS.

THE COUNTRIES OF AUSTRIA-HUNGARY.* Descriptive and Picturesque. CROATIA AND SLAVONIA, by Dr. *Friedrich S. Krauss*. BOSNIA AND HERZEGOVINA, by Dr. *Morits Hoernes*. Vienna: 1889. Karl Græser.

THE two little books, in German, above noted, are the fourteenth and fifteenth volumes of a highly interesting series of works upon the various peoples forming the body of the Austro-Hungarian monarchy. No one of the great nations of civilized Europe, perhaps, is less studied in our country than the checkered realm ruled over by the Hapsburg dynasty; yet no country, whether in its historical and political, or in its social and ethnical features, presents such a diversified and fascinating field for the student of universal history. Here the Orient and the Occident meet. The marks of the ancient world, of Byzantium and of Rome, still stand uneffaced by the side of the broken remnants of modern Ottoman civilization. Slave and Magyar, Teuton and Czech, rub uneasily together in this vast conglomerate of migrated nations. Austria-Hungary is the ground where opposite civilizations join and blend, and deserves, from a social and ethnological point of view, the same attention that its imposing dynastical career has met with from the student of political history.

The volume by Dr. Krauss is especially interesting. The civilization, literature, and folk-lore of the Croatian and Slavonian peoples are the result of unique historical and geographical influences. The customs and religious superstitions of the peasantry are survivals of different epochs and changes in the current of national life—a rich field of research for students of folk-lore. The descriptions of Dr. Krauss are remarkably comprehensive, when we regard the size of the little work. The author belongs to a distinguished Hebrew family of scholars in Central Europe. The work upon Bosnia and Herzegovina is similar in scope and plan to that upon Croatia and Slavonia, having the additional interest of treating a subject not so well-known, and engaging our intention more, perhaps, on account of the recent occupation of these little lands by the forces of the Austrian empire. The late Crown Prince Rudolph took a prominent part in the editing of this series. *μκρκ.*

TURGOT. *Léon Say*. Translated by Melville B. Anderson. A. C. McClurg & Co., Chicago.

THIS is the fifth volume in a series of works, of mingled critical and biographical order, on "The Great French Writers." M. Say's work is written with great sympathy and intelligence, and presents us with a portrait of one of the most interesting characters of the period of the French revolution, one who shared neither the wild hopes of its extreme advocates, nor the condemnatory opinions of more timorous and conservative minds. Turgot is generally described, even by those who admire and wish to do him entire justice, as a failure; his non-success as statesman and economist being ascribed to the troubled condition of the times which would not permit a restless and suspicious populace to accept any leader for a long period. M. Say wishes to make it clear that he does not look upon Turgot in this light, and would "treat him not as defeated but victorious," calling him the founder of our present political economy, meaning, as he says, the French, for it would be difficult to say just what that science is at present in this country, though we may agree with the writer, the present century, being one of industry and "the application of great scientific, geographical, economic discoveries to the development of labor and wealth." Turgot did great service to the general cause of social progress by his teachings respecting the worth of labor.

C. P. W.

* Die Länder Oesterreich-Ungarn's in Wort und Bild. *Kroatien* und *Slavonien*. *Bosnien* und *Herzegovina*.

DIE DEUTSCHEN VOLKSLIEDER UND MÆRCHEN. Zwei Vorträge von *Karl Knortz*. Zürich: 1889. Verlags-Magazin.

Karl Knortz, the well known German translator of American poetry, dedicates these two lectures on German popular songs to German-Americans who, he hopes, will be good American citizens but at the same time preserve and cherish the literary treasures of their fatherland. It is their mission, he says, to cling to the humanitarian ideals of their great poets. An appendix contains about twelve German translations of popular ballads of Yorkshire.

The *Art Amateur* for February is especially full of information of a technical character which must be very useful to young amateurs. Mr. Kellogg continues his letter to a young China painter, which are very full and suggestive, and there are four articles of interest to Amateur Photographers.

"My Note Book" has various entertaining bits, but the most striking is an anecdote of Verestchagin, who threw a packet of cigars presented to him into the sea, exclaiming, "I wont smoke. If we can get art and fashion arranged against this vile practice, there may be some hope of its overthrow, at least in polite circles."

Greta's Boston Art Notes are very lively and give such a tempting description of Bach Bay Park, that one is inclined to hire a buggy immediately to explore its beauties. It may be safe to wait until June to realize all its charms.

The Designs for this number are very pretty. The Moonlight Landscape is soft and pleasing, if not very strong, and the Fern Decoration for a Tea Service is quite worthy of reproduction.

A design of Lilies for a Border is original and well conventionalized, and the decoration of Barnacles for a fish plate is suggestive and appropriate, perhaps, if not altogether agreeable. The motto "First Fruits of them that Sleep," does not strike us as appropriate to the design of wheat sheaves, as farmers usually have to be up pretty early, to secure a good crop.

The Magazine sustains its well earned reputation and does a good work in directing the thoughts and studies of young America. We should be glad to see more solid historical and critical matter in it. E. D. C.

NOTES.

Brentano's have issued a new book entitled "Frederick Struther's Romance," by Albert Ulmann.

Prof. Felix Adler lectured at the Chicago Opera House, on Sunday last, upon "The Education of Children." Prof. Adler proposes the inductive method, leading from practical examples to higher principles.

The discussion between Col. Ingersoll, Frederick R. Coudert, and Gov. Stewart L. Woodford upon "The Limits of Toleration," held before the Nineteenth Century Club, will be published in the *Truth Seeker* of Feb. 16th.

"The Last Journal" of the late Lady Brassey will be published here at once by Longmans, Green & Co. It contains an account of the trip of the "Sunbeam" to India, Borneo, and Australia. The publishers have sought to make this one of the most sumptuous volumes of late years.

We have received from the publishing house of Hermann Riemann of Berlin, copies of several pamphlets in a series entitled "Allgemein-verständliche Naturwissenschaftliche Abhandlungen," essays on scientific subjects, in popular language. The treatise by Dr. V. Schlegel on "The So-called Four-dimensional Space" is thoughtful and suggestive.

"It is a matter of record," says Governor Oglesby, in his biennial message to the General Assembly of Illinois, "that the labors of the State Board of Health towards the prevention of epidemics of contagious and infectious diseases, have resulted in a saving of nearly $3,500,000 to the people of the commonwealth in 1881 and 1882, when smallpox was epidemic."